Estimation of temperature-dependent growth profiles of fly larvae with application to criminology

Frédéric Ferraty

(*Joint work with D. Pigoli, J. A. D. Aston, A. Mazumder, C. Richards, M.J.R. Hall*)

Prague, July 2018

Outside crime scene

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- **post-mortem time** (pmi)

  =

  time difference between death and discovery of corpses

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- Important issue : **How estimating pmi beyond a few days ?**

Solution : forensic entomology

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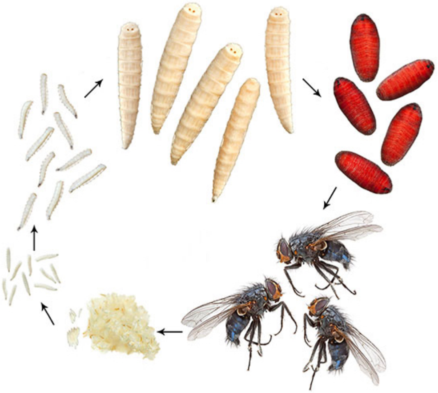

BLOWFLY LIFE CYCLE
(calliphora vicina)

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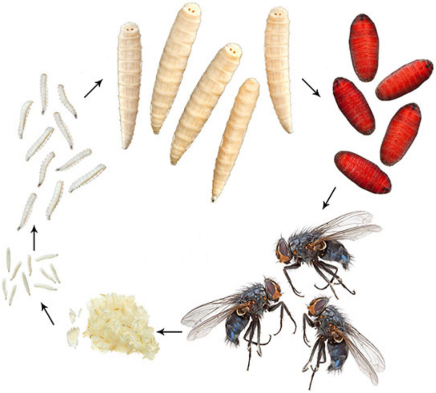

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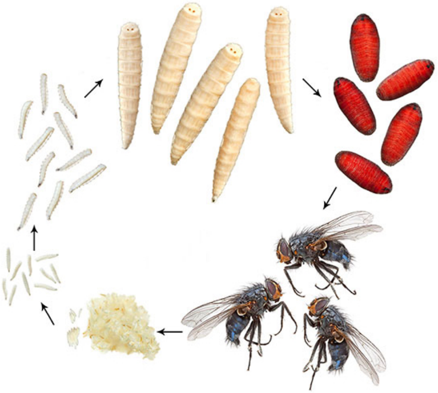

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- instar larvae (with two moults)

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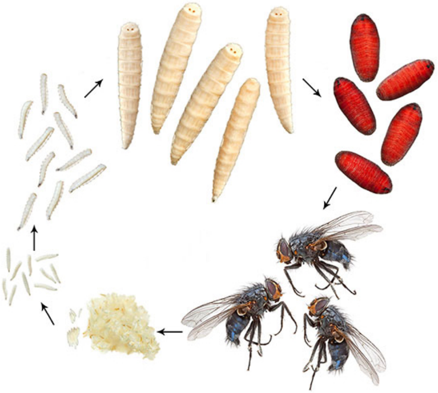

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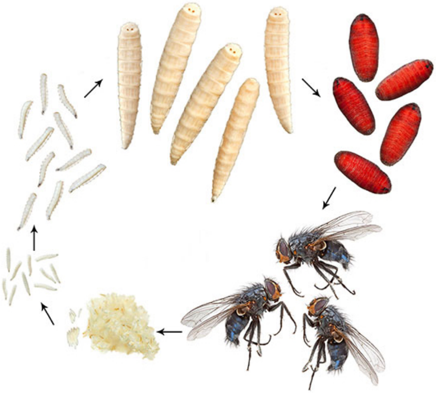

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- eggs
- instar larvae (with two moults)
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- adult fly

Current investigator approach for estimating *pmi*

Accumulated Degree Hours (ADH) between t_0 and t_1

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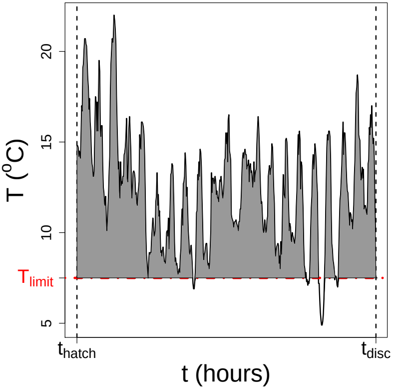

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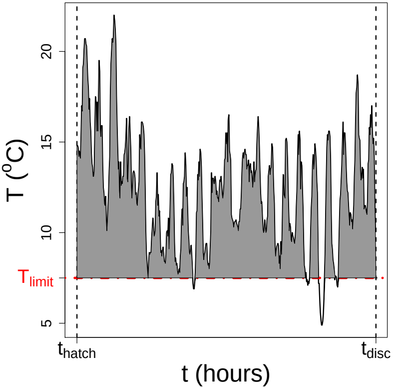

Interpretation

ADH = amount of thermal energy available to the larva

Current investigator approach for estimating *pmi*

Numerical approximation (step = 1 hour)

$ADH \approx pmi \times \{\overline{T} - T_{limit}\}$ where $\overline{T}$ = averaged temperatures

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experimental results about ADH w.r.t. developmental stage of larvae

sample of larvae at most advanced stage collected from the corpse

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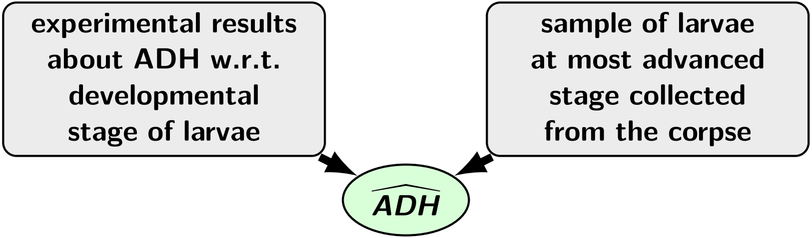

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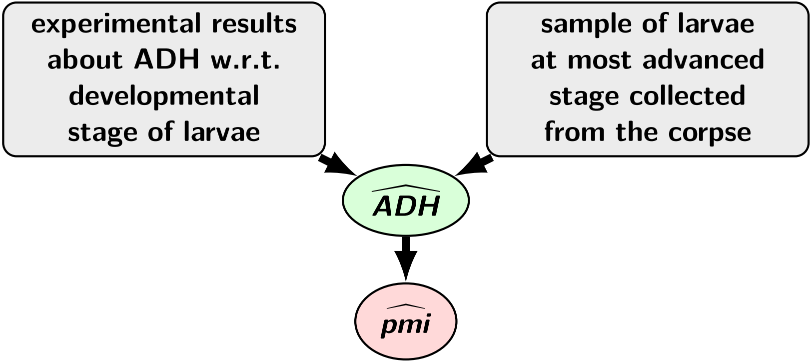

Growth profile approach

Estimating maggot age from its growth profile

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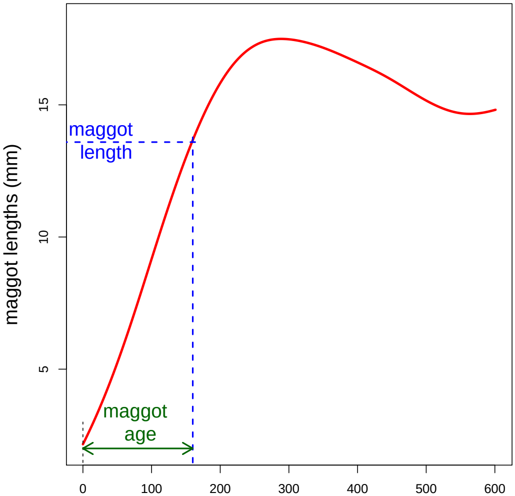
maggot growth profile



maggot length
(collected from the corpse)

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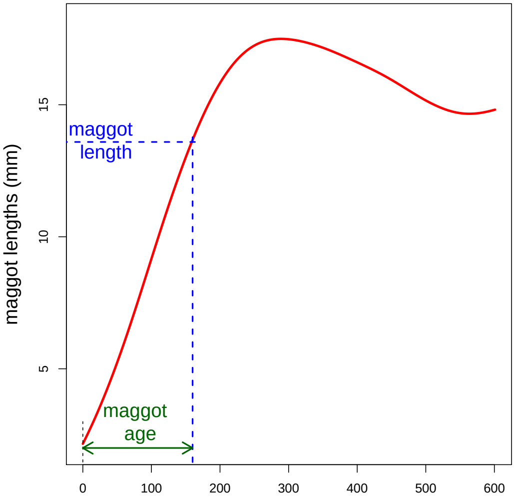

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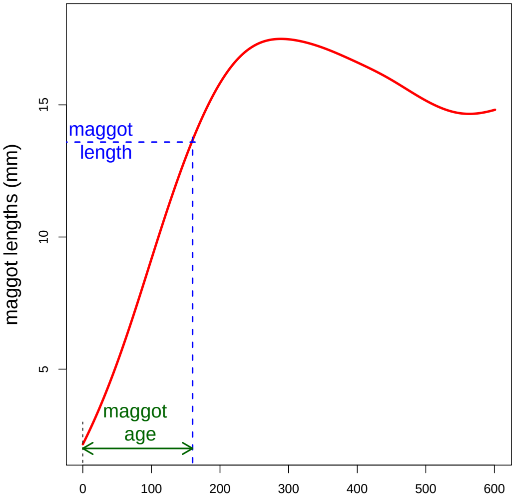

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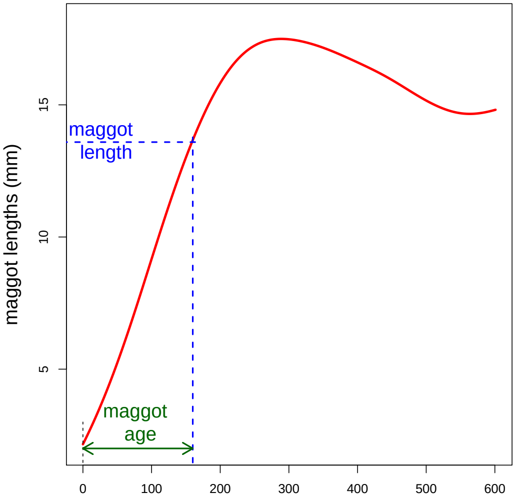

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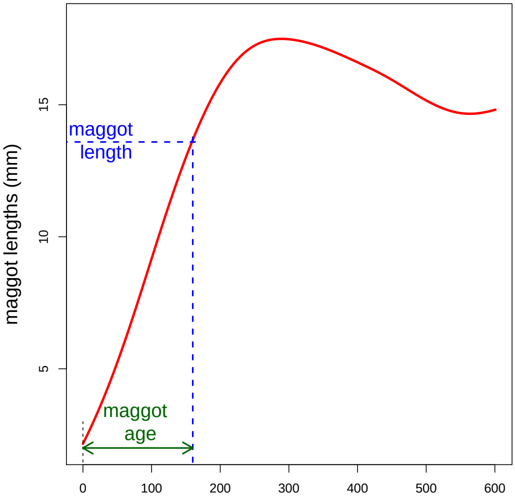

maggot length
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Growth profile approach

Assumption

pmi $\approx$ *oldest* larva (at the most advanced developmental stage)

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pmi $\approx$ *oldest* larva (at the most advanced developmental stage)

To determine pmi one has to

1. estimate the length of the *oldest* larva
2. estimate the growth profile

Available data

Crime scene data

Investigators

1. collect larvae from the body at the most advanced developmental stage
   $\hookrightarrow$ larval lengths $Y_1^*, Y_2^*, \ldots, Y_{n_{obs}}^*$
2. record "continuously" temperatures at the crime scene

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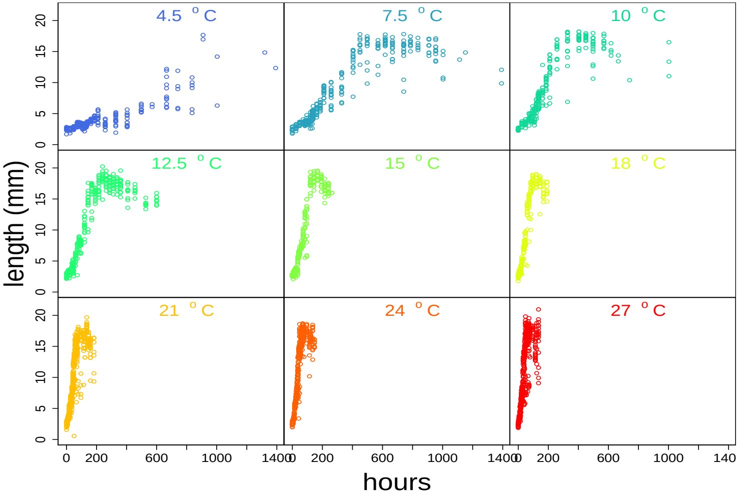
Additional data

- temperatures from the closest wheather station
  $\hookrightarrow$ recover past temperatures at the crime scene
- Experimental developmental larvae data

Available data

Experimental developmental larvae data depends on temperature (blowfly calliphora vicina)



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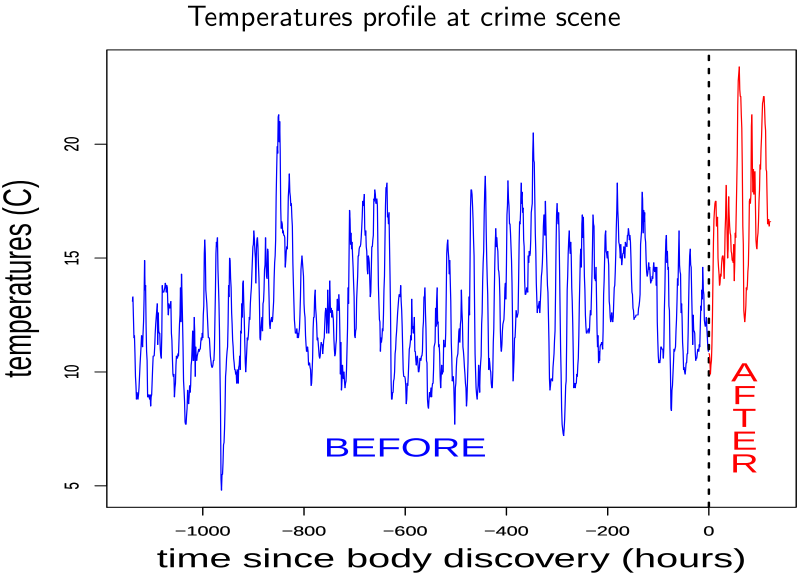
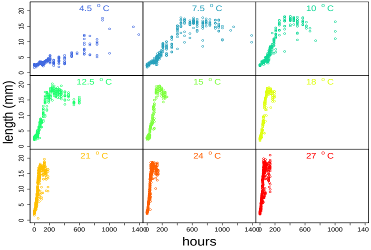


Statistical challenge ?

Constant temperature growth data



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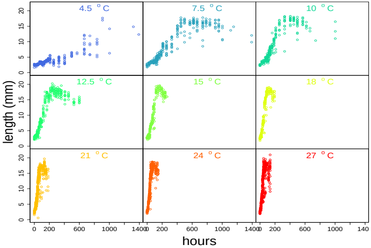

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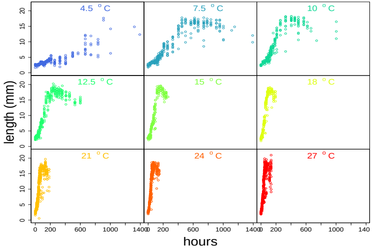

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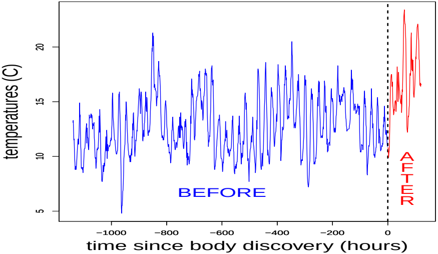
Temperature profile at crime scene



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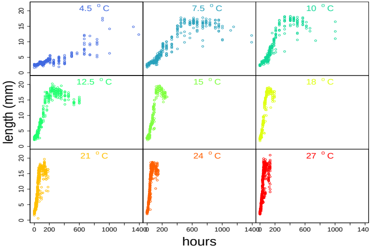

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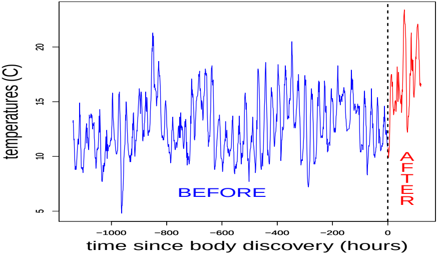

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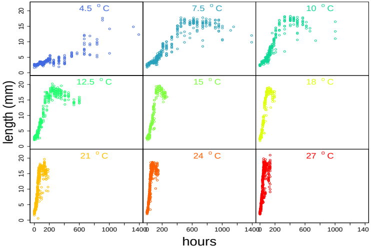

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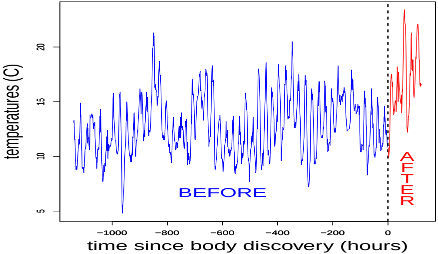

Varying temperature growth profile

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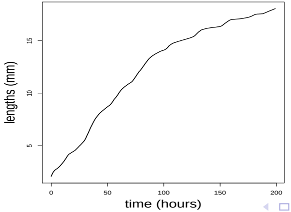

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Four-steps estimating procedure

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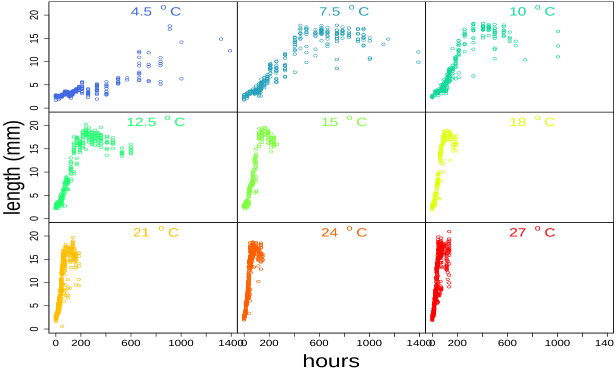
4. Build the varying temperature growth profile from a dynamic model and estimate pmi

Step 1 : smoothing experimental developmental data



- $T_1 < T_2 < \cdots < T_K$: K experimental temperatures

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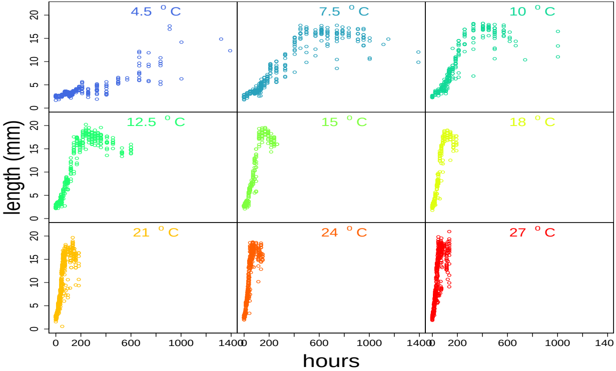

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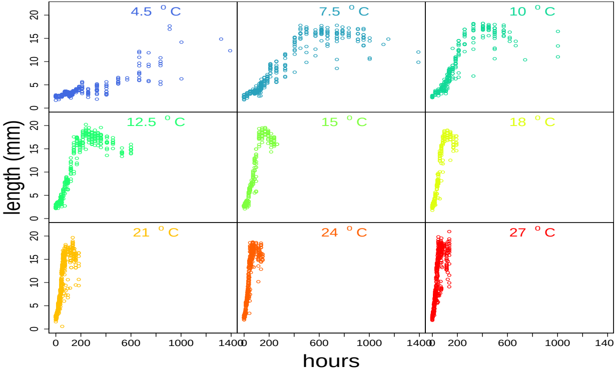

- $T_1 < T_2 < \cdots < T_K$: K experimental temperatures
- $0 = t_1^k < t_2^k < t_{n_k}^k := t_{pup}^k$: observation times at temperature T_k
- $Y_{kj1}, Y_{kj2}, \ldots, Y_{kjN_{kj}}$: N_{kj} observed larval lengths at time t_j^k after hatching and exposed at temperature T_k

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$L_{T_k}(t)$:= benchmark growth profile at temperature T_k and time after hatching t

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Nonparametric regression model

$$Y_{kj\ell} = L_{T_k}(t_j^k) + \varepsilon_{kj\ell} \quad (k = 1, \ldots, K,\ j = 1, \ldots, n_k,\ \ell = 1, \ldots, N_{kj})$$

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Local linear regression

$$Q(a, b) := \sum_{j=1}^{n_k} \omega_k(t_j^k) \left\{ \overline{Y}_{kj} - a - b(t_j^k - t) \right\}^2 K \left\{ h_L^{-1}(t_j^k - t) \right\}$$

with $\overline{Y}_{kj} := N_{kj}^{-1} \sum_\ell Y_{kj\ell}$ and $\omega_k(t_j^k) = N_{kj} / \sum_j N_{kj}$

$$\boldsymbol{\left\{ \widetilde{L}_{T_k}(t), \widetilde{L}'_{T_k}(t) \right\} := \arg\min_{a,b} Q(a, b)}$$

Step 1 : smoothing experimental developmental data

Notations

- $\boldsymbol{X}_t = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ t_1^k - t & t_2^k - t & \cdots & t_{n_k}^k - t \end{pmatrix}^T$

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Local linear estimators

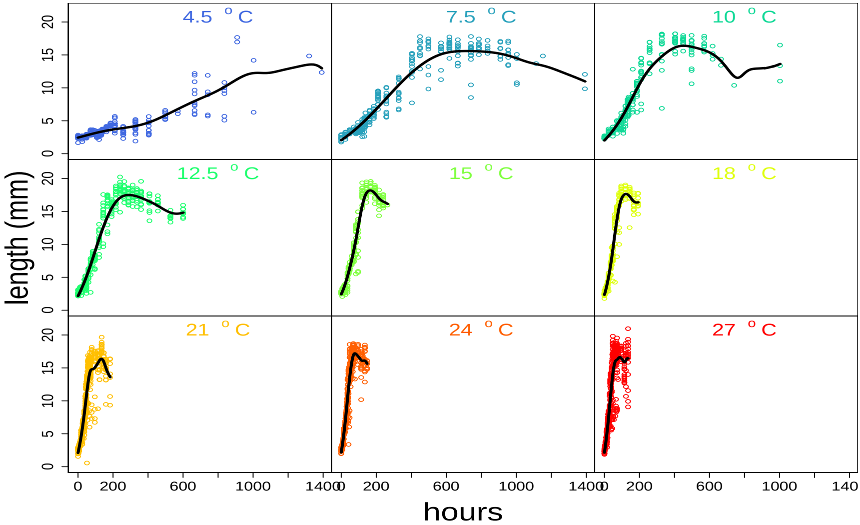
$$\widetilde{L}_{T_k}(t) = (1,0)^T \left(\boldsymbol{X}_t^T \boldsymbol{K}_t \boldsymbol{X}_t\right)^{-1} \boldsymbol{X}_t^T \boldsymbol{K}_t \overline{\boldsymbol{Y}^k}$$

and

$$\widetilde{L}'_{T_k}(t) = (0,1)^T \left(\boldsymbol{X}_t^T \boldsymbol{K}_t \boldsymbol{X}_t\right)^{-1} \boldsymbol{X}_t^T \boldsymbol{K}_t \overline{\boldsymbol{Y}^k}.$$

Step 1 : smoothing experimental developmental data

benchmark growth profiles $\widetilde{L}_{T_1}, \ldots, \widetilde{L}_{T_K}$



Step 1 : asymptotics

(H1) $\forall T$, L_T four-times continuously differentiable with a nonnull second derivative in the neighbourhood of the maximum length time

(H2) $n := \inf_k n_k$: $n \to \infty$, $h_L \to 0$ with n, $n\,h_L^6 \to \infty$ with n

Theorem 1

Under (H1)-(H2) + standard ones, for any $T_k \in \{T_1, \ldots, T_K\}$

$$\left\| \widetilde{L}_{T_k} - L_{T_k} \right\|_\infty = O(h_L^2) + O_P\left\{ (nh_L^3)^{-1/3} \right\}$$

and

$$\left\| \widetilde{L'}_{T_k} - L'_{T_k} \right\|_\infty = O(h_L^2) + O_P\left\{ (nh_L^6)^{-1/3} \right\}$$

Step 2 : growth profile = shape profile ∘ time warping

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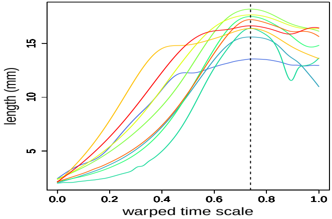
bench. shape profiles
$\widetilde{S}_{T_1}, \ldots, \widetilde{S}_{T_K}$



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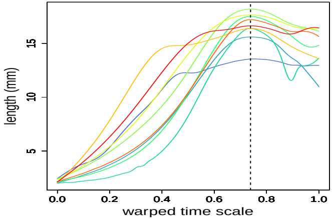

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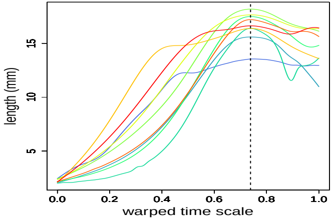

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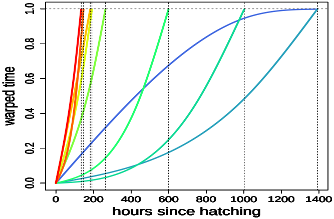
bench. warping functions

$\widetilde{w}_{T_1}, \ldots, \widetilde{w}_{T_K}$



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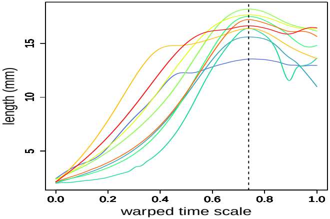

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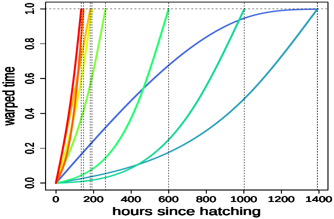

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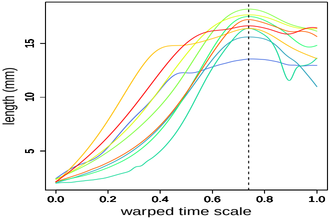
bench. shape profiles

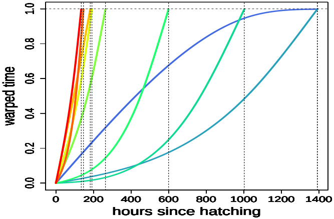
$\widetilde{S}_{T_1}, \dots, \widetilde{S}_{T_K}$



$\circ$

bench. warping functions

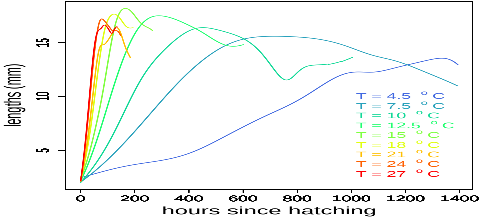
$\widetilde{w}_{T_1}, \dots, \widetilde{w}_{T_K}$



benchmark growth profiles

$$\widetilde{L}_{T_1} = \widetilde{S}_{T_1} \circ \widetilde{w}_{T_1}, \dots, \widetilde{L}_{T_K} = \widetilde{S}_{T_K} \circ \widetilde{w}_{T_K}$$

=



Step 2 : benchmark warping functions (landmark registration)

Warping function assumptions

1. $w_{T_1}, \ldots, w_{T_K}$ align
   - hatching time $t_{hatch} = 0$
   - time of maximum length t_{max}
   - pupation time t_{pup} (largest time after hatching)
2. $w_{T_1}, \ldots, w_{T_K}$ are strictly increasing quadratic polynomial

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$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

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$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$w_{T_1}, \ldots, w_{T_K}$ are well defined (existence and unicity)

Step 2 : estimation of benchmark warping functions

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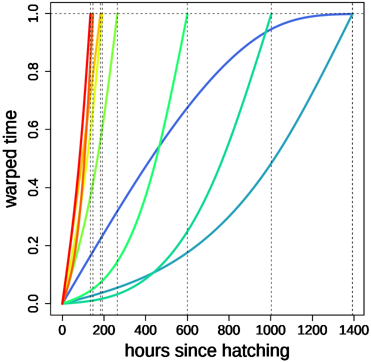
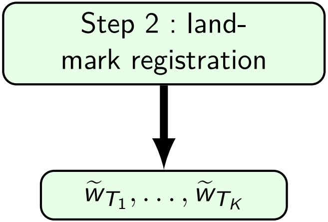


Step 2 : estimation of benchmark shape profiles



Step 2 : estimation of benchmark shape profiles

Step 2 : landmark registration

$\downarrow$

$\widetilde{w}_{T_1}, \ldots, \widetilde{w}_{T_K}$

Step1 : smoothing experimental developmental data

$\downarrow$

$\widetilde{L}_{T_1}, \ldots, \widetilde{L}_{T_K}$

Step 2 : estimation of benchmark shape profiles

Step 2 : landmark registration

Step1 : smoothing experimental developmental data

$\widetilde{w}_{T_1}, \ldots, \widetilde{w}_{T_K}$

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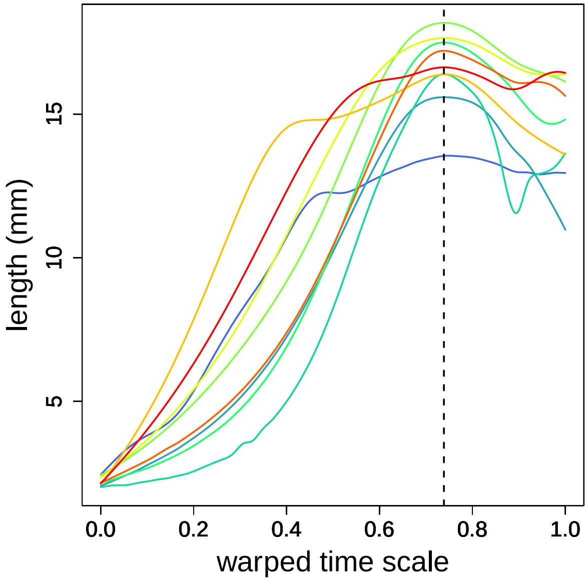

Step 2 : Asymptotics (shape profiles and warping func.)

Theorem 2

$\forall T_k \in \{T_1, \ldots, T_K\}$, as soon as L_{T_k} and S_{T_k} regular enough

$$\|\widetilde{w}_{T_k} - w_{T_k}\|_\infty = O(h_L^2) + O_P\left\{\left(nh_L^6\right)^{-1/3}\right\} = \|\widetilde{S}_{T_k} - S_{T_k}\|_\infty$$

and the same rate holds for $\|\widetilde{w}'_{T_k} - w'_{T_k}\|_\infty$ and $\|\widetilde{S}'_{T_k} - S'_{T_k}\|_\infty$

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Remark

$\widetilde{w}_{T_k}$ involves $\widetilde{t}^k_{max}$ estimation of t^k_{max} where $\widetilde{t}^k_{max}$ (resp. t^k_{max}) is such that $\widetilde{L}'_{T_k}(\widetilde{t}^k_{max}) = 0$ (resp. $L'_{T_k}(t^k_{max}) = 0$)

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$\hookrightarrow$ $\widetilde{w}_{T_k}$ inherits asymptotic property of $\widetilde{L}'_{T_k}$

$\hookrightarrow$ $\widetilde{S}_{T_k} \overset{def}{=} \widetilde{L}_{T_k} \circ \widetilde{w}_{T_k}^{-1}$ inherits asymptotic property of $\widetilde{L}'_{T_k}$

Step 3 : **growth profile** at any temperature T

Step 2

$$(\widetilde{S}_{T_1}, T_1), \ldots, (\widetilde{S}_{T_K}, T_K)$$

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$$\widehat{S}_T(u) = \frac{\sum_{k=1}^{K} \widetilde{S}_{T_k}(u)\, K_S\left\{h_S^{-1}(T_k - T)\right\}}{\sum_{k=1}^{K} K_S\left\{h_S^{-1}(T_k - T)\right\}}$$

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$$\widehat{w}_T(t) = \frac{\sum_{k=1}^{K} \widetilde{w}_{T_k}(t)\, K_w\left\{h_w^{-1}(T_k - T)\right\}}{\sum_{k=1}^{K} K_w\left\{h_w^{-1}(T_k - T)\right\}}$$

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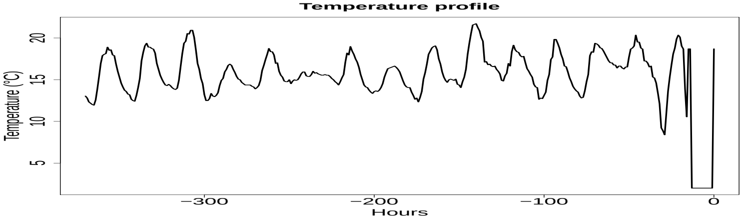
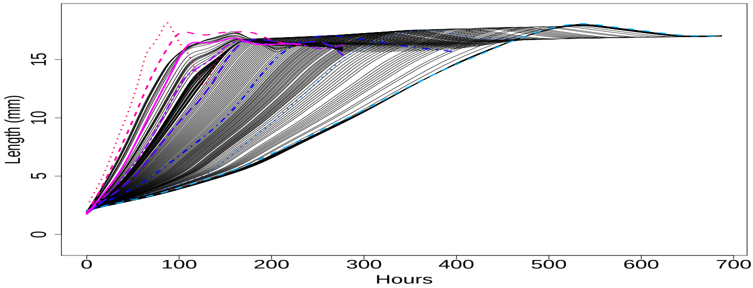
$$\widehat{L}_T(t) = \widehat{S}_T \circ \widehat{w}_T(t)$$

Step3 : growth profile at any temperature T



Growth profiles for any temperature at crime scene



Step 3 : **growth profile derivative** at any temp. T

Step 2

$(\widetilde{S}'_{T_1}, T_1), \ldots, (\widetilde{S}'_{T_K}, T_K)$

Step 3 : **growth profile derivative** at any temp. T

Step 2

$$(\widetilde{S}'_{T_1}, T_1), \dots, (\widetilde{S}'_{T_K}, T_K)$$

$$\widehat{S}'_T(u) \quad = \frac{\sum_{k=1}^K \widetilde{S}'_{T_k}(u)\, K_{S'}\left\{h_{S'}^{-1}(T_k - T)\right\}}{\sum_{k=1}^K K_{S'}\left\{h_{S'}^{-1}(T_k - T)\right\}}$$

Step 3 : **growth profile derivative** at any temp. T

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$$\widehat{L}'_T = \left\{\widehat{S}'_T \circ \widehat{w}_T(t)\right\} \widehat{w}'_T(t)$$

Step 3 : Asymptotics (growth profile at any temp. T)

Theorem 3

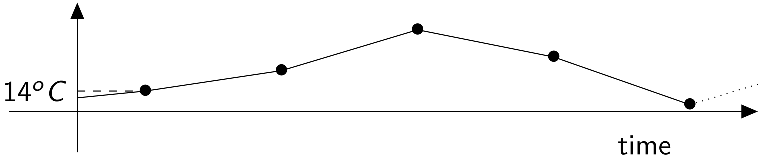
For any temperature T

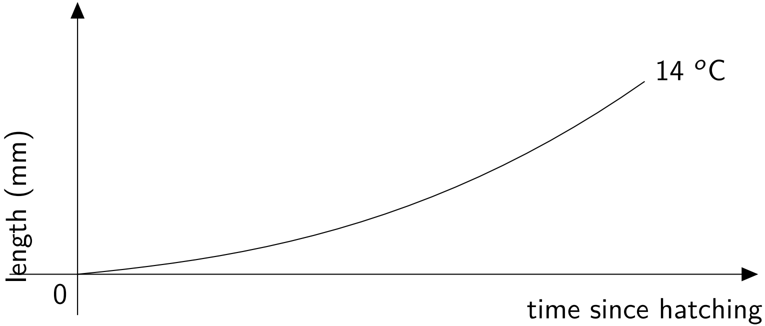
$$\left\|\widehat{L}_T - L_T\right\|_\infty = O(h_L^2) + O(h_S) + O(h_w) + O_P\left\{\left(n\,h_L^6\right)^{-1/3}\right\}$$

and

$$\left\|\widehat{L}'_T - L'_T\right\|_\infty = O(h_L^2) + O(h_{S'}) + O(h_w) + O(h_{w'}) + O_P\left\{(nh_L^6)^{-1/3}\right\}$$

Step 4 : varying temperature growth profile





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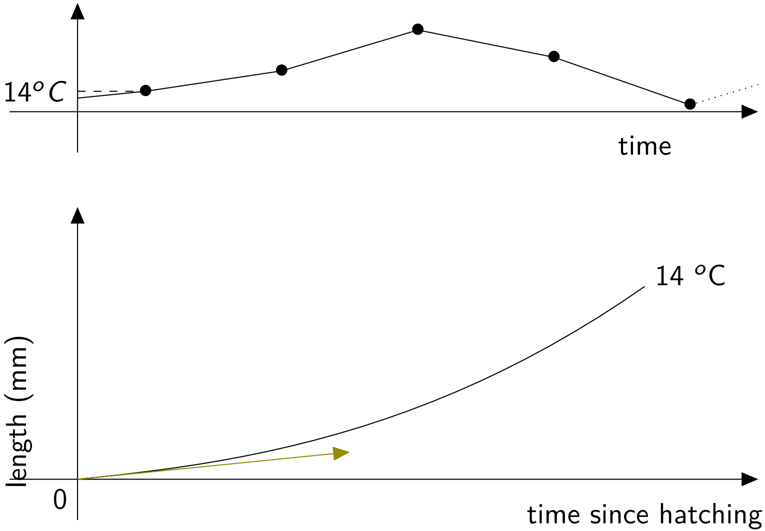

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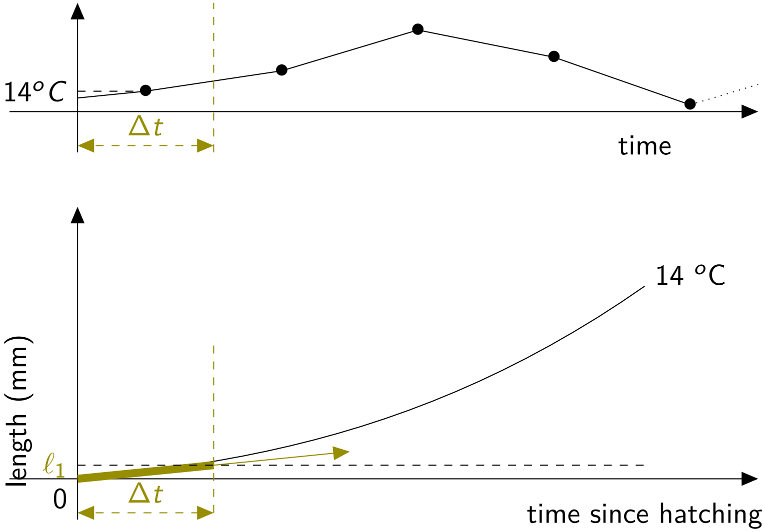

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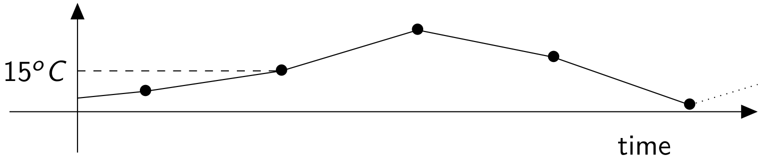

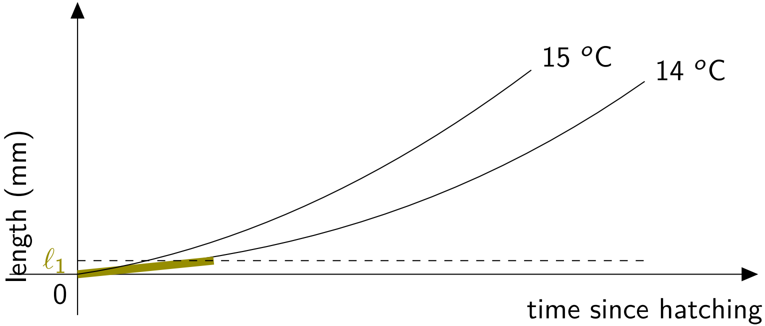

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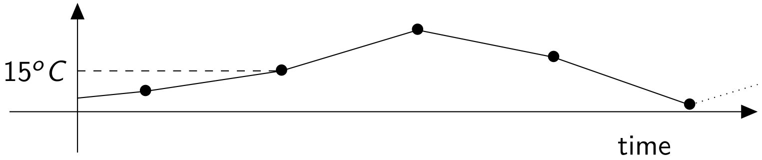

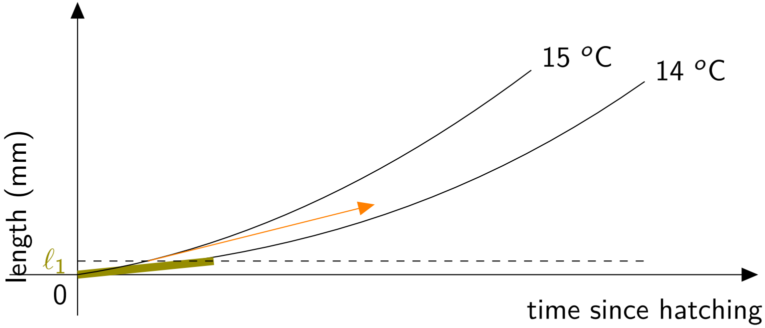

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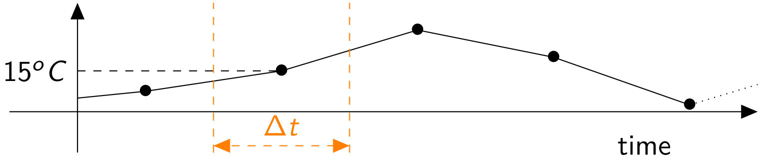

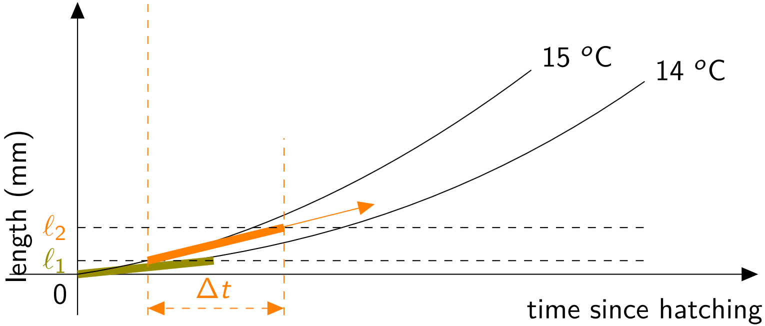

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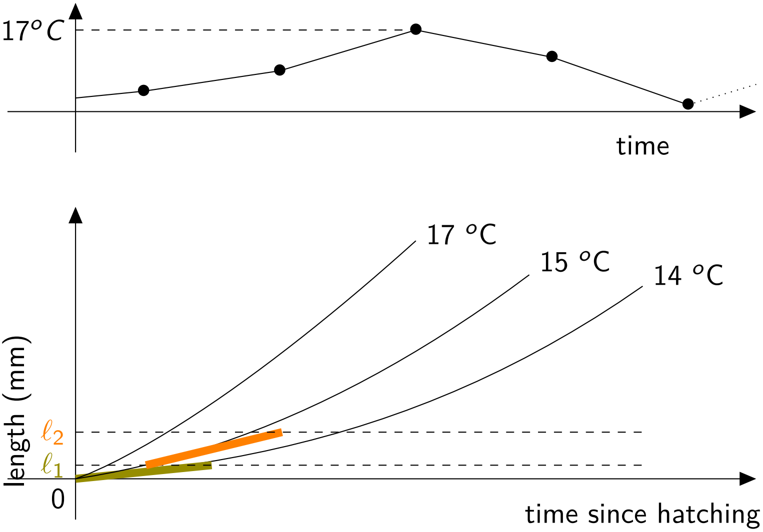

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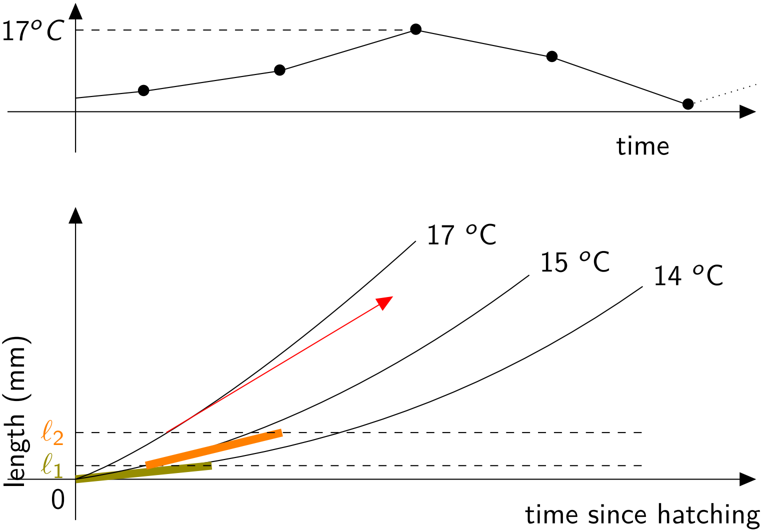

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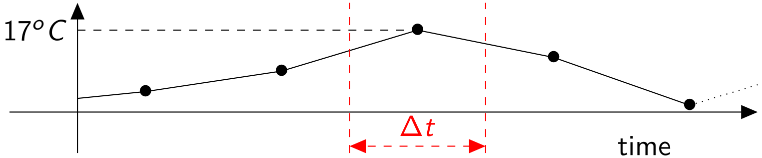

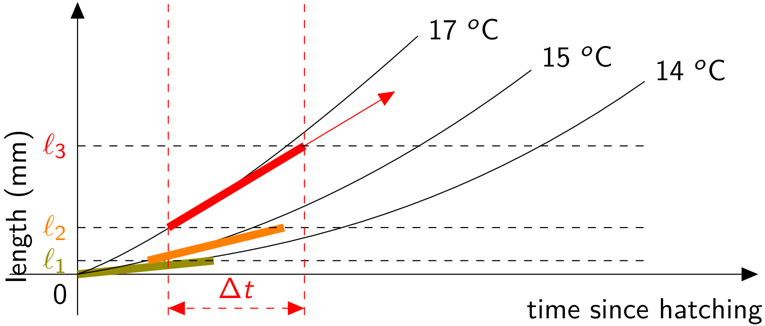

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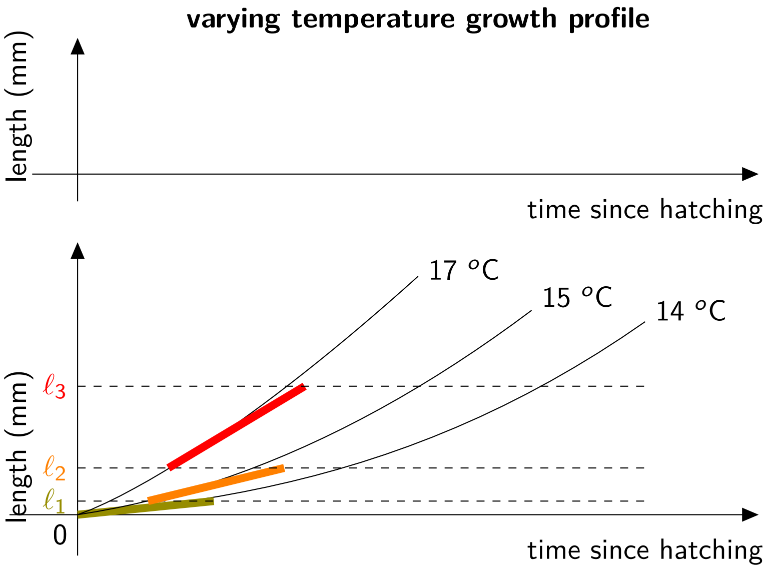

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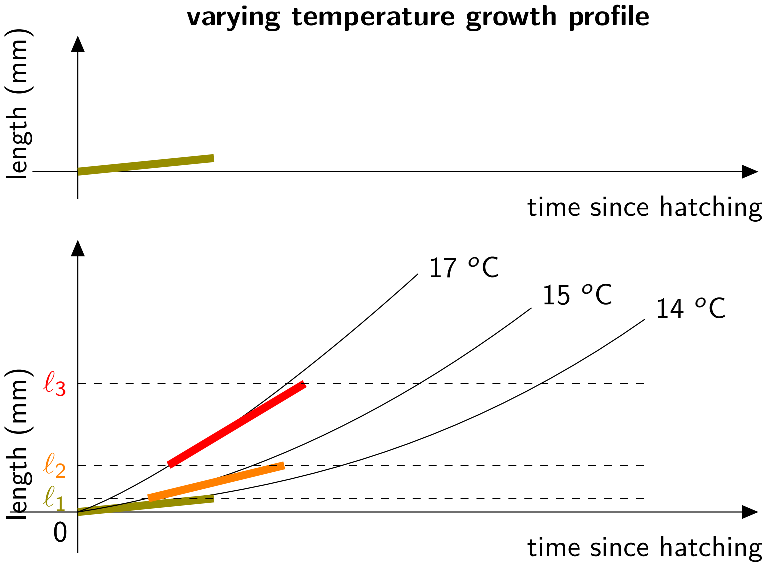

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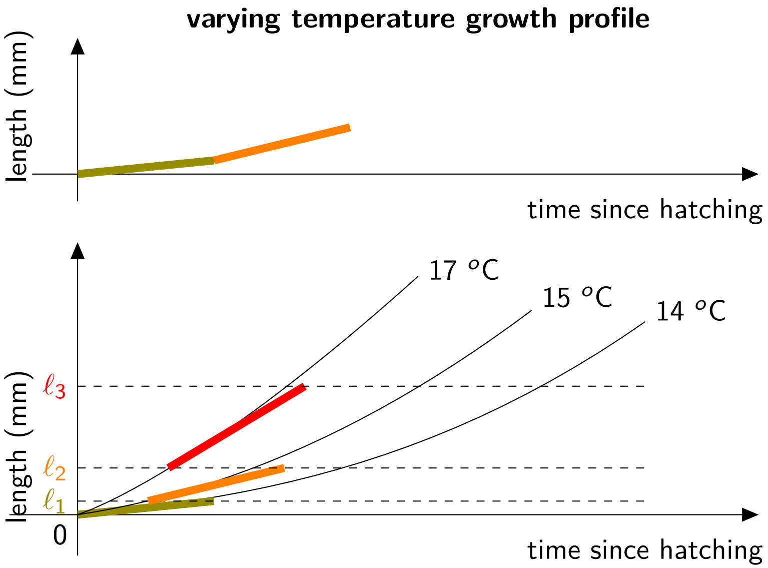

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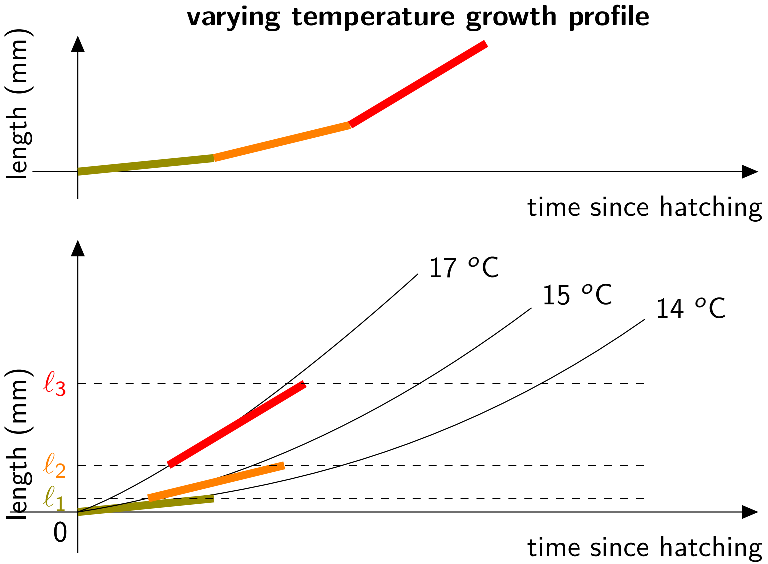

Step 4 : dynamic growth model

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- $\{T_v,\ v \in [0,t)\}$ = temperature variation up t
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- $\{T_v,\ v \in [0, t)\}$ = temperature variation up t
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- the growth process follows the local dynamics of the correspondent constant temperature growth curve at the point of the curve which reaches the current length

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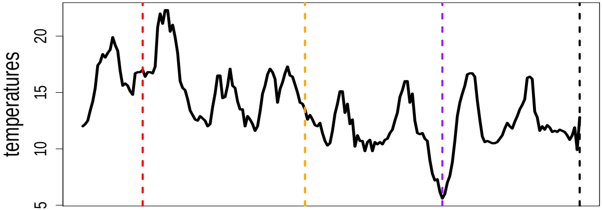
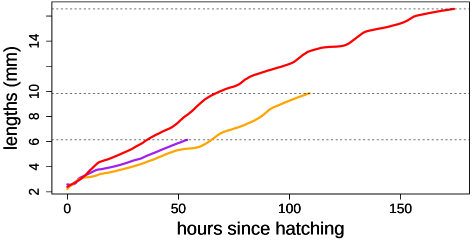
temperature profile at crime scene



-175 -110 -54 $t_{disc} = 0$

(hours before corpse discovery)

estimated varying temperature growth profile



Step 4 : Asymptotics (varying temp. growth profile)

Theorem 4

$$\left\|\widehat{L} - L\right\|_\infty = O(h_L^2) + O(h_S) + O(h_{S'}) + O(h_w) + O(h_{w'}) + O(p^{-1}) + O_P\left\{(nh_L^6)^{-1/3}\right\}$$

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Interpretation. Given average length $\overline{Y}$, the estimated hatching time $\widehat{t}_{hatch}$ is s.t. $\widehat{L}(t^* - \widehat{t}_{hatch}) \approx \overline{Y}$

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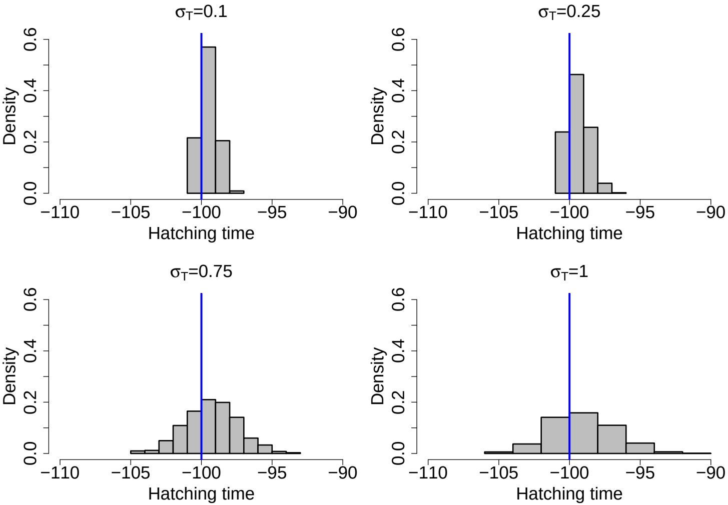
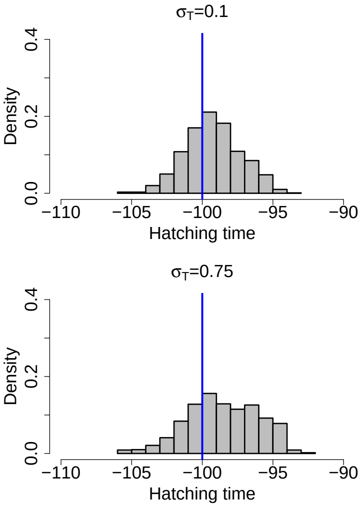
- $\widetilde{L}_{10^o} =$ estimated varying growth length at constant temp 10^{o}C
- simulate 1000 samples of 20 larvae lengths $Y_1^*, \ldots, Y_{20}^*$ s.t. $Y_1^* = \widehat{L}(100) + \varepsilon_1, \ldots, Y_{20}^* = \widehat{L}(100) + \varepsilon_{20}$ with iid errors

Constant temp. profile → hatching time distribution



Varying temp. profile → hatching time distribution



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No external corroboration (such as defendant confession) of the time the body has been abandoned

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$\hookrightarrow$ compare our Growth Profile Approach (GPA) with ADH method currenly used

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- $t_{hatch} \in (-371, 0)$ considered reasonable by forensic scientists
- 70 *Calliphora vicina* larvae collected from the body

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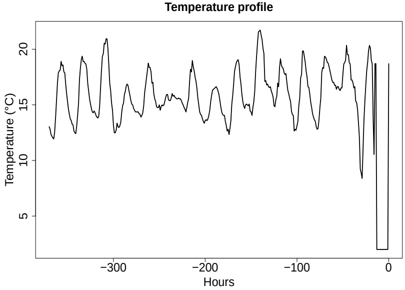

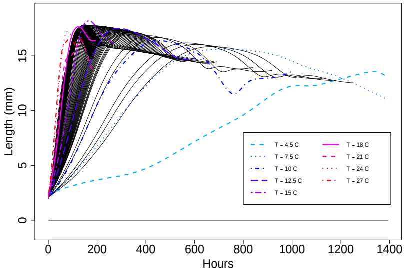

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95% confidence interval

$\widehat{pmi}_{ADH}$	$\widehat{pmi}_{GPA}$
$[-276, -228]$	$[-260, -190]$

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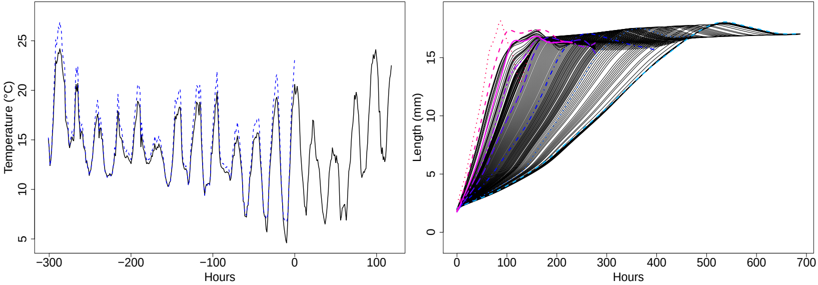

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$\widehat{pmi}_{ADH}$	$\widehat{pmi}_{GPA}$
$[-270, -240]$	$[-255, -249]$

Thank you for your attention !